

ASSIGNMENT 2: ASYMPTOTIC NOTATION

Instructor: Mehmet Koyutürk and Orhan Özgüner

Due: February 2 before 11:59 PM

Problem 1

For the following statements, consider the functions $f(n)$, $g(n)$ and constant c such that $f(n) \geq 0$, $g(n) \geq 0$, and $c > 0$. Indicate whether the statements are true or false. If true prove the statement by providing a formal argument based on the definition of asymptotic notation, otherwise, provide a counter-example to prove that they are false.

$$(a) \max\{f(n), g(n)\} = \Theta(f(n) + g(n)).$$

$$(b_1) f(n) + c = O(f(n)).$$

$$(b_2) \text{ If } f(n) \geq 1, \text{ then } f(n) + c = O(f(n)).$$

$$(c_1) \text{ If } f(n) = O(g(n)), \log(f(n)) \geq 0 \text{ and } \log(g(n)) \geq 0, \text{ then } \log(f(n)) = O(\log(g(n))).$$

$$(c_2) \text{ If } f(n) = O(g(n)), \log(f(n)) \geq 0 \text{ and } \log(g(n)) \geq 1, \text{ then } \log(f(n)) = O(\log(g(n))).$$

$$(d_1) f(2n) = \Theta(f(n)).$$

$$(d_2) \text{ If } f(n) = O(n^c), \text{ then } f(2n) = O(n^c).$$

$$(d_3) \text{ If } f(n) = \Theta(n^c), \text{ then } f(2n) = \Theta(f(n)).$$

Problem 2

Assume ϵ , a and b are constants such that $0 < \epsilon < 1 < a < b$. Sort the following functions in asymptotically increasing order and indicate when two or more functions are asymptotically equivalent (see definition below). Briefly justify your answers (no formal proof is needed).

$$n^\epsilon$$

$$\epsilon^n$$

$$a^n$$

$$b^n$$

$$a^{\log_a n}$$

$$\log(n^a)$$

$$\log(n^b)$$

$$n/a$$

$$\epsilon n$$

$$(n+a)^b$$

$$n^{a+b}$$

$$(n+b)^a$$

$$n^{-\epsilon}$$

$$n^{-a}$$

$$n^a$$

$$\log(n^\epsilon)$$

$$\log_{1/\epsilon} n$$

$$(\log n)^a$$

$$\log(bn)$$

$$a^{\epsilon n}$$

Definition. Two functions $f(n)$ and $g(n)$ are **asymptotically equivalent** if and only if $f(n) = \Theta(g(n))$.

Example. Suppose we are sorting the following functions: n , $\log(n)$, $2n$, 2^n . Then, the asymptotical ordering is: $\log(n) \ll n \equiv 2n \ll 2^n$.