Homework 4

HW Notes:

- Box your final answer. You will be graded on both the final answer and the steps leading to it. Correct intermediate steps will help earn partial credit.
- For full credit, ~~cross out~~ any incorrect intermediate steps.
- If you need to make any additional assumptions, state them clearly.
- Legible writing will help when it comes to partial credit.
- Simplify your result when possible.

Problems:

1. [10] Use the table of FT pairs and the table of properties to find the FT of each of the following signals (*DO NOT USE INTEGRATION*):

   (a) $x(t) = 2\text{rect}\left(\frac{t-2}{4}\right)$

   (b) $x(t) = e^{-3t}\text{rect}\left(\frac{t-2}{4}\right)$

   (c) $x(t) = t\text{rect}\left(\frac{t-2}{4}\right)$

   (d) $x(t) = \cos(4\pi t)\text{rect}\left(\frac{t-2}{4}\right)$

2. [5] Find a mathematical expression and sketch or plot the inverse FT of $F(\omega) = \text{sinc}^3(\omega/2)$. *Hint*: the inverse FT formula would probably be a hard way to do it.

3. [5] Find the FT of $t^2 e^{-(t/2)^2}$. *Hint*: see table of FT pairs.

4. [5] Show that if $f(t)$ is real and odd, then $F(\omega)$ is purely imaginary and odd.

5. [5] Consider a real signal $f(t)$ and let

$$f(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega), \quad F(\omega) = \text{real}\{F(\omega)\} + j\ \text{imag}\{F(\omega)\}$$

   and

$$f(t) = f_e(t) + f_o(t)$$

   where $f_e(t)$ and $f_o(t)$ are the even and odd component of $f(t)$ respectively. Show that

$$f_e(t) \overset{\mathcal{F}}{\longleftrightarrow} \text{real}\{F(\omega)\} \qquad\qquad f_o(t) \overset{\mathcal{F}}{\longleftrightarrow} j\ \text{imag}\{F(\omega)\}$$

6. [5] Find the energy of the signal $x(t) = t\text{sinc}^2(t)$ by Fourier methods.

7. [5] What percentage of the total energy in the energy signal $f(t) = e^{-t}u(t)$ is contained in the frequency band $-7\text{rad/s} \leq \omega \leq 7\text{rad/s}$.

8. [10] A LTI system has the following frequency response:

$$H(j\omega) = \frac{-\omega^2 + j\omega + 1}{(-\omega^2 + 6j\omega + 25)(j\omega + 2)}.$$

   (a) [10] Find the impulse response of the LTI system.
   Hint: first find the partial differential equation.

(b) [10] Find the differential equation corresponding to the LTI system.
Hint: write $H(\omega) = Y(\omega)/X(\omega)$ and cross multiply.

9. [10] Find the FT of the following signal: $x(t) = \sum_{n=-\infty}^{\infty} 2\delta(t-6n) - \delta(t-6n-2) - \delta(t-6n+2)$. sketch the magnitude of the spectrum.

10. [10] Compute the Fourier transform of each of the following signals

(a) $[e^{-\alpha t}\cos\omega_0 t]u(t), \alpha > 0$

(b) $e^{-3|t|}\sin 2t$

11. [10] Determine the continuous-time signal corresponding to the following transform.

(a) $X(j\omega) = cos(4\omega + \pi/3)$

(b) $X(j\omega)$ as given by magnitude and phase plots.

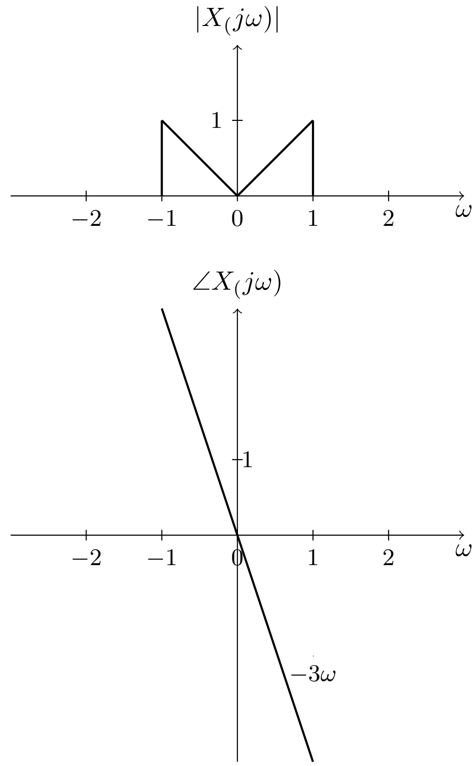

Figure: 0402

12. [10] Shown in the figure 0403 is the frequency response $H(j\omega)$ of a continuous-time filter referred to as a lowpass differentiator. For each of the input signals x(t) below, determine the filter output signal $y(t)$.

(a) $x(t) = \cos(2\pi t + \theta)$

(b) $x(t) = \cos(4\pi t + \theta)$

(c) $x(t)$ is a half-wave rectified sine wave of period 1, as sketched in figure 0404.

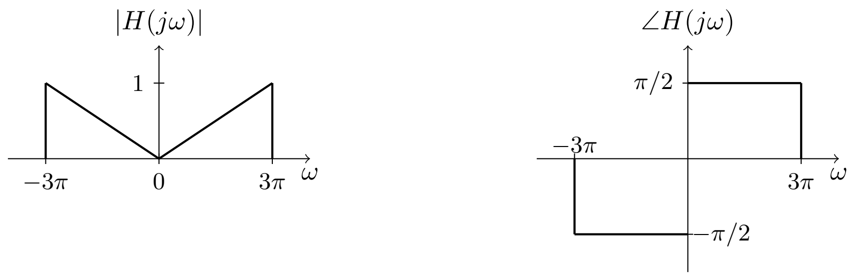

Figure: 0403

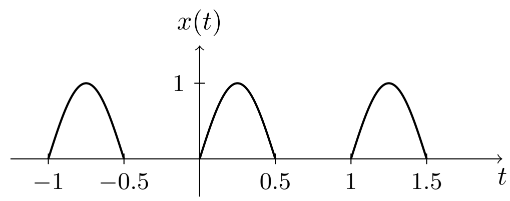

Figure: 0404

$$x(t) = \begin{cases} \sin(2\pi t) & , m \le t \le m + \frac{1}{2} \\ 0 & , (m + \frac{1}{2}) \le t \le m \end{cases}$$

13. [10] A power signal with the power spectral density shown in figure 0405 is the input of a linear system with the frequency response shown in figure 0406. Calculate and sketch the power spectral density of the system's output signal.

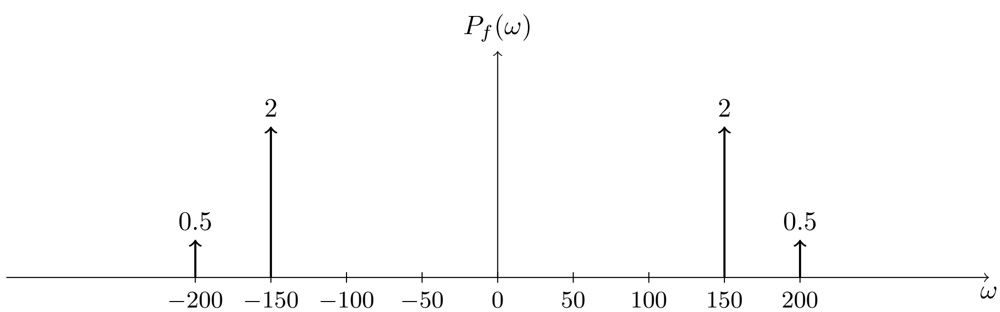

Figure: 0405

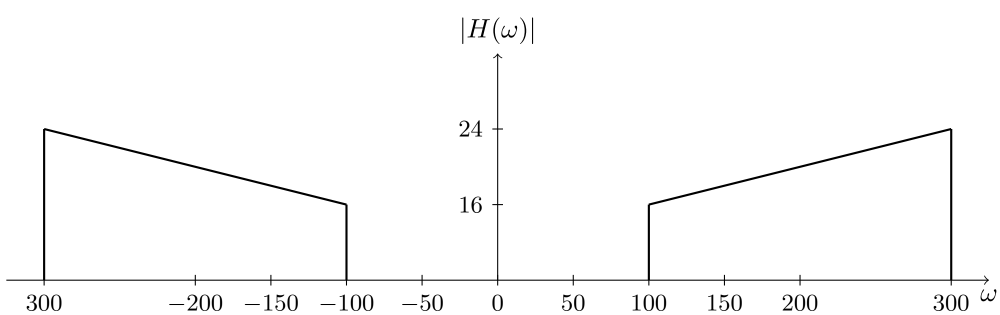

Figure: 0406