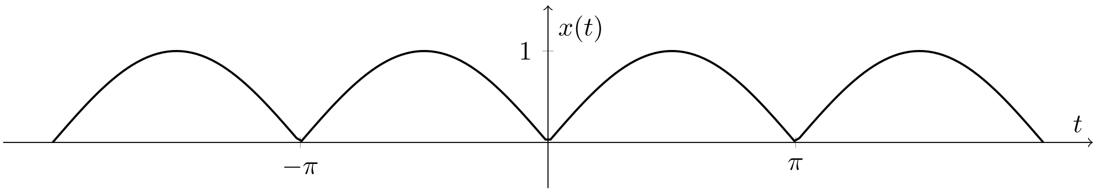
Homework 1

HW Notes:

- Box your final answer.
- If you need to make any additional assumptions, state them clearly.
- Simplify your result when possible.
- For the problems with [credit!], no partial credit will be given if the final answer is wrong.

Problems:

1. [6!] Consider the periodic sinusoidal signal illustrated below.



   (a) Find the mathematical representation for this signal.

   (b) Find the energy of this signal. Is it an energy signal, power signal, or neither?

   (c) Carefully sketch and find a mathematical expression for the output signal of an integrator system, i.e., $y(t) = \int_{-\infty}^{t} x(\tau)\mathrm{d}\tau$ where $x(t)$ is under the interval $[-\pi, \pi]$.

2. [12!] Determine the values of average power and energy for each of the following signals:

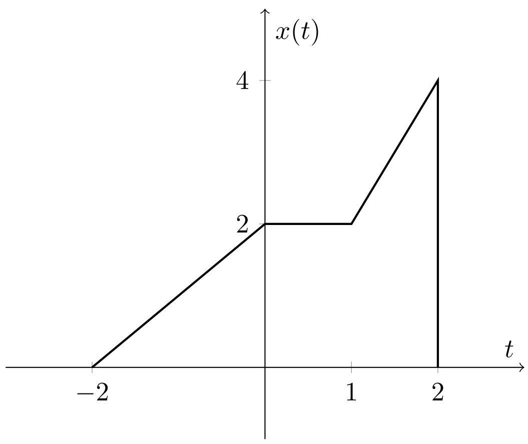
   (a) $x_1(t) = e^{-2t}u(t)$

   (b) $x_2(t) = e^{j\left(2t+\frac{\pi}{4}\right)}$

   (c) $x_3(t) = \cos(t)$

3. [6!] Find the average value, power, and energy of signal $x(t) = \begin{cases} e^{-t} & t > 0 \\ 0 & \text{otherwise} \end{cases}$

4. [6!] Consider the signal illustrated below



   (a) Find a mathematical representation for $x(t)$.

(b) Sketch $s(t) = \frac{1}{2}x(-2t+1)$ by performing graphical time transformations. Sketch the intermediate signal each time you make a transformation, like time-shifting, or time-scaling.

(c) Decompose $x(t)$ into its even and odd components. Carefully sketch the even and odd components of $x(t)$.

5. [9!] Suppose $x_1(t)$ and $x_2(t)$ are periodic signals with fundamental periods $T_1 > 0$ and $T_2 > 0$ respectively.

   (a) Show that if T_1/T_2 is rational, then $x(t) = x_1(t) + x_2(t)$ is periodic.

   (b) Similarly, show that if T_1/T_2 is rational, then $x(t) = x_1(t)x_2(t)$ is periodic and the least common multiple of T_1 and T_2 is a period of $x(t)$.

   (c) Determine whether the following signals are periodic. If so, find a period. Otherwise, specify the reason.

      i. $x(t) = \sin\left(\frac{\pi t}{3}\right)\cos\left(\frac{\pi t}{4}\right) + \sin\left(\frac{\pi t}{5}\right)\sin\left(\frac{\pi t}{2}\right)$

      ii. $x(t) = \sin\left(\frac{\sqrt{3}}{3}\pi t\right) + \sin\left(\frac{\pi t}{5}\right)$

6. [4!] Considering the signals

$$x(t) = \cos\left(\frac{2}{3}\pi t\right) + 2\sin\left(\frac{16}{3}\pi t\right)$$
$$y(t) = \sin(\pi t)$$

Show that $z(t) = x(t)y(t)$ is periodic, and write $z(t)$ as a linear combination of harmonically related complex exponentials. That is, find a number T and complex numbers C_k such that

$$z(t) = \sum_k c_k e^{jk\left(\frac{2\pi}{T}\right)t}$$

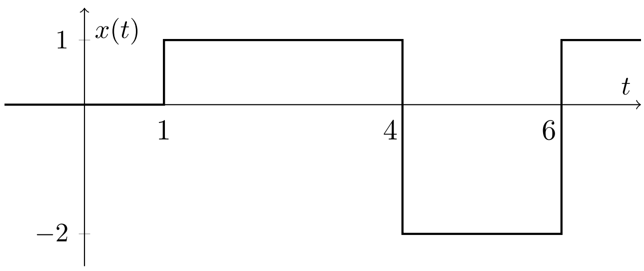
7. [6!] Use MATLAB to plot the following three signals.

   (a) $y(t) = e^t$

   (b) $y(t) = e^{-\frac{t}{10}}\sin(\pi t)$

   (c) $y(t) = \sin\left(\pi t + \frac{\pi}{4}\right)$

8. [4!] Consider the signal illustrated below.



   (a) Express the signal $x(t)$ using a sum of step functions.

   (b) Find the derivative of the signal and carefully sketch it.

9. [15!] Indicate whether the following systems are Memoryless, Time Invariant, Linear, Causal, Stable. Justify your answers. (3! for each)

   (a) $y(t) = x(t-2) + x(2-t)$

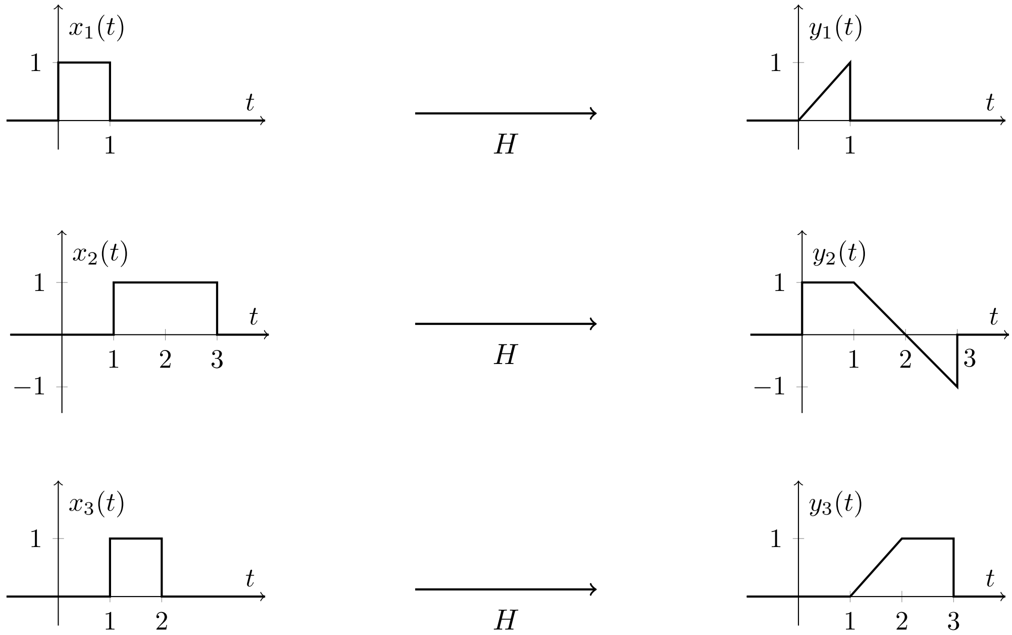
   (b) $y(t) = x\left(\frac{t}{3}\right)$

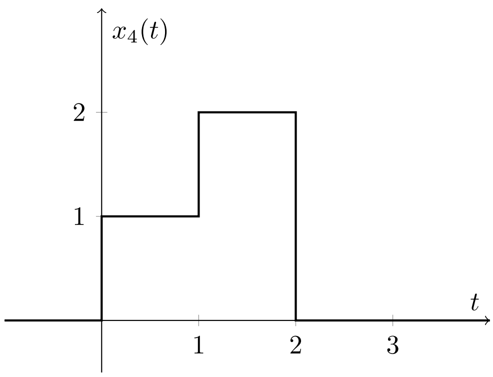
   (c) $y(t) = \cos(x(t))$

   (d) $y(t) = \int_{-\infty}^{\frac{t}{2}} x(\tau)\mathrm{d}\tau$

   (e) $y(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t}$

10. [8!] A linear system H has following input-output pairs. Answer the following question, and justify your answers.





   (a) Could this system be causal?

   (b) Could this system be time invariant?

   (c) Could this system be memoryless?

   (d) What is the output for the input $x_4(t)$? Sketch it.

11. [4!] Prove that the product of two odd signals is an even signal.

12. Let $s(t) = \left(\frac{t-1}{2}\right)^2 \text{rect}\left(\frac{t-1}{2}\right)$

    (a) Make a sketch of $s(t)$.

    (b) Evaluate $\int_{-\infty}^{\infty} s(t)x(t)\mathrm{d}t$, where $x(t) = \delta\left(t - \frac{1}{2}\right) + \delta(t-2) - \delta(3t-4)$

13. [5!] Show that causality for a continuous-time linear system is equivalent to the following statement: For any time t_0 and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

14. [5!] A system has the input and output relation given by

$$y(t) = tx(t).$$

    Is the system

    (a) linear?

    (b) time invariant?

    (c) bounded input bounded output (BIBO) stable?

    (d) memoryless?

    (e) causal?

15. [4!] Given a signal $x(t)$,

    (a) suppose it is an energy signal with energy $E[x(t)] = E_x$. Then what is the energy of the signal $x(-at+b)$, i.e., $E[x(-at+b)]$?

    (b) suppose it is a power signal with power $P[x(t)] = P_x$. Then what is the power of the signal $x(-at+b)$, i.e., $P[x(-at+b)]$?