CFRM 405: Mathematical Methods for Quantitative Finance Homework 1

Instructions: Solve each of the exercises both by hand (must show all work for credit) and using Mathematica to verify that you get the same answers. You do not need to provide your Mathematica code.

1. Compute the following limits.

   (a) $\lim_{h\to 0} \dfrac{5(x+h-3)^2 - 5(x-3)^2}{h}$ Also, this limit is the derivative for what function?

   (b) $\lim_{x\to\infty} \dfrac{x}{\sqrt{4x^4 - 2x^3 - 10x^2 + 2x}}$

2. Determine the x value of the critical points of the following functions. Determine if each critical point corresponds to a local minimum or local maximum.

   (a) $f(x) = 2\cos(x) - x$

   (b) $f(x) = 1 + 80x^3 + 5x^4 - 2x^5$

3. Compute the derivatives of the following functions.

   (a) $f(x) = \dfrac{1}{(1-x)^2}$

   (b) $f(x) = \displaystyle\sum_{k=1}^{20} k e^{-a_k x^4}$ (the $\{a_k\}$ are constants)

   (c) $f(x) = \dfrac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{x^2}{2}\right)(T-t)}{x\sqrt{T-t}}$ ($S > 0$, $K > 0$, r, q, and $T > t$ constant)

   (d) $f(x) = \dfrac{\log\left(\frac{S}{K}\right) + \left(x - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ ($S > 0$, $K > 0$, q, $\sigma > 0$, and $T > t$ constant)

   (e) $f(x) = \dfrac{\log\left(\frac{x}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ ($K > 0$, r, q, $\sigma > 0$, and $T > t$ constant)

4. Recall that

$$d_+(\cdot) = \frac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

   (a) Parts (c), (d), and (e) of Problem 2 correspond to partial derivatives of d_+. What partial derivative does each correspond to?

   (b) Compute the partial derivative of d_+ with respect to t.

5. Compute the following antiderivatives.

   (a) $\displaystyle\int x^2 e^x \, dx$

   (b) $\displaystyle\int \left[\log(x)\right]^2 dx$

   (c) $\displaystyle\int x^2 \log(x) \, dx$

6. Evaluate the following definite integrals.

   (a) $\displaystyle\int_2^6 x^2 \log(x) \, dx$

   (b) $\displaystyle\int_0^4 \frac{1}{(1+x)^2} \, dx$

7. If you are an online student, please write down the proctor you are using for the exams in this class. They must be approved (or pending approval) at the time of submission of this assignment. The guidelines and procedure for proctor approval can be found at: http://depts.washington.edu/compfin/info/exam-proctors/