EE6203 HW1 ASSIGNMENT By Dr Lee PH

Instructions :

Solutions in PDF format only must be uploaded to NTULearn EE6203 "Assignments" folder by 19 OCT 21 TUESDAY 8.00 am.

All detailed workings are to be shown as marks will be awarded accordingly. Only hand-written solutions are accepted. Typed solutions will not be accepted. Write your name clearly on every page of your solutions submitted.

1. Scan your work with a mobile phone/tablet scanner apps. Combined all into **ONE** pdf file to be submitted.
2. Please name the file to be submitted with your matriculated FULL NAME.
3. Ensure all images are clear and **readable** before submitting.
4. Submission through email is **NOT** acceptable.
5. **Solutions must be uploaded to NTULearn EE6203 "Assignments" folder by 19 OCT 21 TUESDAY 8.00 am.**
6. Zero marks will be given to a student who did not submit the assignment by the due date/time without a valid reason.
7. You may use the Transform Tables as posted to the course-site in the folder "Lee PH".
8. There are 5 questions in this HW1. Answer all questions.

1. A system is described by the following difference equation

$$c(k+3)+3c(k+2)+5c(k+1)+7c(k)=9u(k)$$

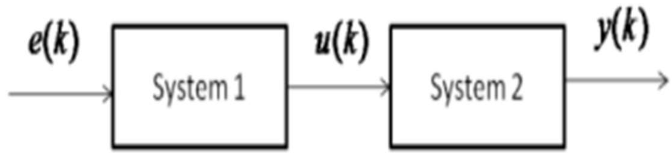
where the output $y(k)=c(k)$. Define the state variables as

$$x_1(k)=c(k); x_2(k)=c(k+1); x_3(k)=c(k+2)$$

Obtain a state-space representation for the system.

(10 marks)

2. Consider the two systems connected as shown below.



The respective state-space representations are given as follow:

System S_1 :
$$0.1u(k+1)+0.2u(k)=0.3e(k)$$

System S_2 :
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.4 & 0.5 \\ 0.6 & 0.7 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.8 \\ 0.9 \end{bmatrix} u(k)$$
$$y(k)=\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

If $x_3(k)=u(k)$, give a state-space representation for the overall system,

$$\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k)+\mathbf{B}e(k); \qquad \mathbf{x}(k)=\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$
$$y(k)=\mathbf{C}\mathbf{x}(k)+de(k)$$

(15 marks)

3. Given the state equation of a linear system as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

The ZOH equivalent, with a sampling period of T seconds, is of the following form:

$$\mathbf{x}(k+1) = \mathbf{A}_d\mathbf{x}(k) + \mathbf{B}_d u(k)$$

If

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; T = 0.5 \text{ sec}$$

(i) Find $\mathbf{A}_d$ and $\mathbf{B}_d$.

(ii) Find the transfer function $\dfrac{\mathbf{X}(z)}{U(z)}$.

(iii) Determine the characteristic equation of the discretised system and obtain the eigenvalues of $\mathbf{A}_d$.

(30 marks)

4. A discrete-time system is given by

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 3 \\ 4 \end{bmatrix}u(k)$$
$$y(k) = \begin{bmatrix} 5 & 6 \end{bmatrix}\mathbf{x}(k)$$

(a) Determine a co-ordinate transformation, i.e. find $\mathbf{Q}$ in the following

$$\mathbf{w}_{\mathbf{Q}}(k) = \mathbf{Q}^{-1}\mathbf{x}(k)$$

that transforms the system into the observable canonical form (OCF). Hence, using $\mathbf{Q}$, determine a state-space representation which is in the OCF form.

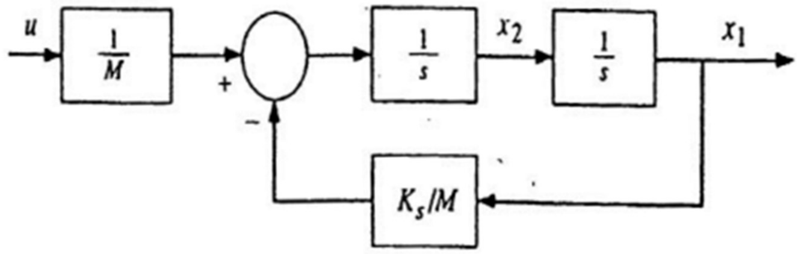
(b) Determine a co-ordinate transformation, i.e. find $\mathbf{P}$ in the following

$$\mathbf{w}_{\mathbf{P}}(k) = \mathbf{P}^{-1}\mathbf{x}(k)$$

that transforms the system into the controllable canonical form (CCF). Hence, using $\mathbf{P}$, determine a state-space representation which is in the CCF form.

(20 marks)

5. A continuous-time system is as shown below. Let $M = 625, K_S = 10$.



(a) Obtain a state-space representation for the continuous-time system with the state variables as indicated and the output variable $y(t) = x_1(t)$.

(b) The system is sampled with a zero-order hold and the sampling period is 0.5 second. Obtain a zero-order hold equivalent of the continuous-time system.

(c) Find the deadbeat control law of the following form

$$u(k) = -\mathbf{Kx}(k)$$

Show that the response is indeed deadbeat.

(25 marks)