SIT292 LINEAR ALGEBRA 2021
Assignment 2

Due 8 p.m September 10 2021

Assignments to be submitted online as one PDF file (no multiple jpegs)
Assignmens can be handwritten and scanned.

1. Find all the cofactors C_{ij} of

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ -4 & 2 & 2 \end{bmatrix}$$

hence find adj$A$.

(ii) Verify that the adj A you obtained is correct by multiplying it with A.

20 marks

2. Find all numbers α such that the vectors

$$\begin{bmatrix} 5 \\ \alpha \\ 3\alpha \\ 1 \\ 5 \end{bmatrix} \text{ and } \begin{bmatrix} \alpha \\ -\alpha \\ 3\alpha \\ -1 \\ 1 \end{bmatrix} \text{ are orthogonal.}$$

6 marks

3. Use Gaussian elimination to reduce the following system of equations to **row-echelon** form, hence solve for $x_1, \ldots, x_4$. Justify the correctness of your solution using matrix ranks

$$\begin{aligned} 2x_1 - 2x_2 + 2x_3 - 4x_4 &= 2 \\ x_1 + 4x_2 + 8x_3 + 2x_4 &= 5 \\ -x_1 + 9x_2 + 3x_3 - 4x_4 &= 5 \end{aligned}$$

10 marks

4. Use Gaussian elimination to reduce the following system of equations to **row-echelon** form, hence solve for $x_1, \ldots, x_3$. Justify the correctness of your solution using matrix ranks

$$\begin{aligned} 3x_2 + 11x_3 &= 6 \\ x_1 + x_2 + 3x_3 &= 2 \\ 3x_1 - 3x_2 - 13x_3 &= -6 \\ -x_1 + 2x_2 + 8x_3 &= 4 \end{aligned}$$

10 marks

5. Use Gaussian-Jordan elimination to find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

10 marks

6. Find eigenvalues and eigenvectors of the matrices A and B, where

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}.$$

Hint: You can guess one eigenvalue for A, B and use long division of polynomials to find the others. 20 marks

7. Diagonalise the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

10 marks

8. For the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

(a) find the eigenvalues and eigenvectors

(b) determine, if possible, a matrix P so that $P^{-1}AP = B$. If impossible, provide an argument for that. (Hint: Use the method in the Study Guide p.113).

($10 + 4 = 14$ marks)

9. For the following matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$$

(a) find the eigenvalues

(b) for each eigenvalue determine the eigenvector(s)

(c) determine a matrix P so that $B = P^{-1}AP$ is in **triangular** form, and verify that the determinant of B agrees with what you used in (a)

($5 + 5 + 10 = 20$ marks)