SIT292 LINEAR ALGEBRA 2021
Assignment 1

Due 8 p.m., August 13, 2021

Assignments to be submitted online as one PDF file (no multiple jpegs)
Assignments can be handwritten and scanned.

1. Define the sets

$$A = \{1, 2, 3\} \qquad B = \{\{1\}, \{2\}, \{3\}\}$$
$$C = \{1, 2, 3, \{2\}, \{3\}, \{1, 2, 3\}\} \qquad D = \{\{3\}, \{2\}, \{1\}, \{1, 2\}, \{1, 2, 3\}\}.$$

Discuss the validity of the following statements

(explain why some are true and why the others are not true).

(a) $A = B$ (d) $A \in C$ (g) $B \subset D$
(b) $A \subseteq B$ (e) $A \subset D$ (h) $B \in D$
(c) $A \subset C$ (f) $C \subset D$ (i) $A \in D$

18 marks

2 Determine (and explain why) whether the relation R on the set of all dogs is reflexive, symmetric, antisymmetric and/or transitive, where $(a, b) \in R$ if and only if

a) a runs faster than b;

b) a and b have the same fur colour;

c) a ate from the same bowl as b.

12 marks

3. Sets describing intervals of **real** numbers are expressed with brackets and endpoints: a square bracket [ ] if the endpoint is included, a round bracket ( ) if the endpoint is excluded. Set $A = [0, 2)$. Then A is the set of all **real** numbers from 0 to 2, including 1 but not including 2. Define also the sets $B = (-5, 0)$ and $C = [1, 3]$.

   (a) Write as intervals the 3 possible pairwise intersections and the 3 possible unions of sets A, B, C. Name the resulting sets as $D, E, ...$ Do not use different letters to denote the same set.

(b) You have several sets now. Define a relation ρ to be "is a subset of" $\subseteq$, on the set consisting of all sets you obtained. Write down the ordered pairs of this relation and draw the Hasse diagram of this partial ordering.

(c) Does the resulting relation define a lattice? (explain why yes or why no)

(d) What is the least upper bound and the greatest lower bound of the set $\{A, B, C\}$?

20 marks

4. Define the relation ρ on the set $S = \{a, b, c, d, e, f\}$ by

$$\rho = \{(a, a), (b, b), (c, c), (d, d), (f, f)(a, b), (a, c), (c, a),$$
$$(b, c), (c, b), (e, d), (d, f), (e, f), (f, e)\}$$

(a) Draw the directed graph of this relation.

(b) Verify whether this is an equivalence relation. If not, which pairs need to be added to ρ to make it an equivalence relation? Write down its equivalence classes.

10 marks

5. Given the binary relations on the set $A = \{1, 2, 3, 4\}$ defined by:

$$\rho_1 = \{(1, 4), (2, 1), (2, 2), (3, 3), (4, 3)\}$$

and

$$\rho_2 = \{(1, 2), (1, 3), (2, 3), (3, 3), (4, 4)\}$$

determine (construct the ordered pairs) of the composite relations:

(a) ρ_1^2

(b) $\rho_1 \circ \rho_2$

(c) $\rho_2 \circ \rho_1$

(d) $\rho_1 \circ \rho_2 \circ \rho_1$

10 marks

6. (i) Use the properties of determinants (page 72 Study Guide (SG)) first to simplify and then to evaluate the determinants of A and B

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ -1 & -3 & -4 & 1 \\ 1 & 2 & 3 & 2 \\ 0 & 0 & 0 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -3 & -2 & 4 \\ 3 & 2 & 5 & 1 \\ 4 & -1 & 3 & 5 \\ 5 & -4 & 1 & 9 \end{bmatrix}$$

(ii) Using the definition of **rank** of a matrix (3.3.1 P 74 SG), evaluate rank(B).

10 marks

7. (Extensions for higher marks) Calculate the determinants of the following matrices, and then solve for x the equations $Det(A) = 0$, $Det(B) = 0$

$$A = \begin{bmatrix} 2 & x & 0 \\ x & 2 & x \\ 0 & x & 2 \end{bmatrix}, B = \begin{bmatrix} x-1 & 0 & 4 \\ 0 & x+1 & 3 \\ 0 & 3 & x+1 \end{bmatrix}.$$

20 marks

8. Prove that points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are collinear if and only if

$$det\left(\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}\right) = 0.$$

20 marks